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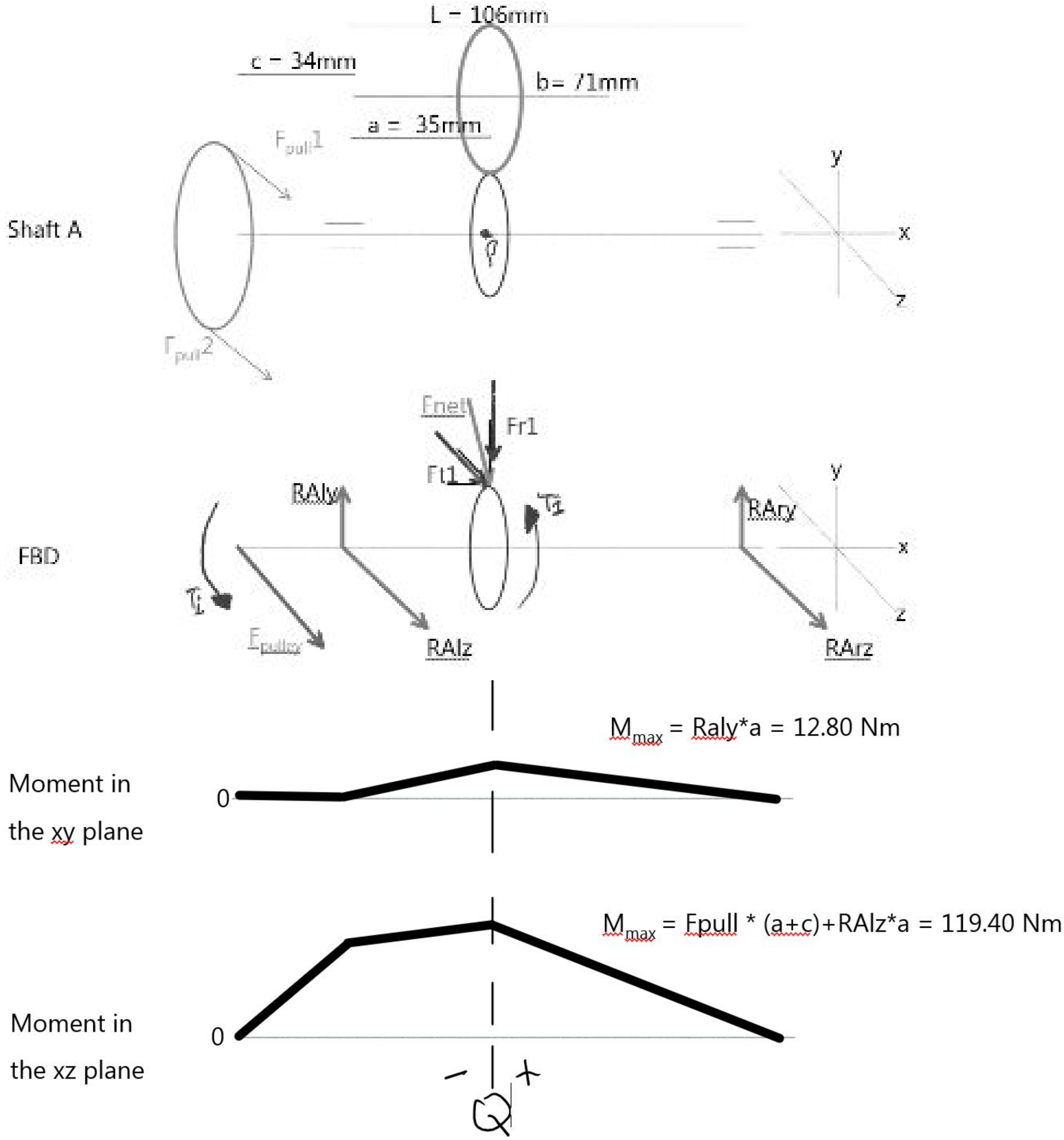
Appendix 1: Shaft System 1 and Related Calculations

1. Shaft A Analysis

a) Shaft A FBD and Bending Moment Diagrams



Q- is the most critical cross section.

b) Analysis of Shaft A forces and moments

Given Parameters:

nA = 2800 RPM, input speed from the motor
P = 26 KW, power input from the motor
Ti = (P * 1000 * 60) / (nA * (2*pi)) = 88.67 Nm , input torque
T1 = -Ti = -88.67 Nm, torque output from gear 1

Shaft Parameters:

The distances a, b, and c were chosen to minimize the moment, while still fitting the size of the various necessary shaft elements within the housing.
a = 35 mm, distance from the center of the left bearing to the center of the gear
b = 71 mm, distance from the center of the left bearing to the center of the right bearing
c = 34 mm, distance from the center of the left bearing to the center of the pulley application element.
dA = 40 mm, diameter of shaft A. This diameter was chosen after running a MATLAB code through increments of basic size until a factor of safety of at least 1.1 was achieved for all relevant criteria
d_adj = 38.4mm, adjusted diameter of shaft A from 4% area reduction due to keyway

Pulley Consideration:

dP = 350 mm, the diameter of the pulley application element. See section 4 for further details
Fpull1/Fpull2 = 1.4 ; the ratio of tension from tight over slack sides of the pulley belt
Fpull2 = 2*Ti/(dP * (Fpull1/Fpull2 - 1)) = 1.266 kN ; tension in the slack side
Fpull1 = 1.5 *Fpull2 = 1.773 kN ; tension in the driving side
Fpull = Fpull1+Fpull2 = 3.040 kN ; total force applied on the shaft from the pulley

Force and Moment Balance:

Initial assumption is that all forces are oriented in the positive direction along their respective axes. See section 3 for more details on naming convention of bearing reaction forces, RA, l/r, y/z.

$$\sum F_x = 0$$

$$\sum F_y = 0 = \mathrm{RAly} + \mathrm{RAry} + \mathrm{Fr1}$$

$$\sum F_z = 0 = \mathrm{RAlz} + \mathrm{Rarz} + \mathrm{Fpulley} + \mathrm{Ft1}$$

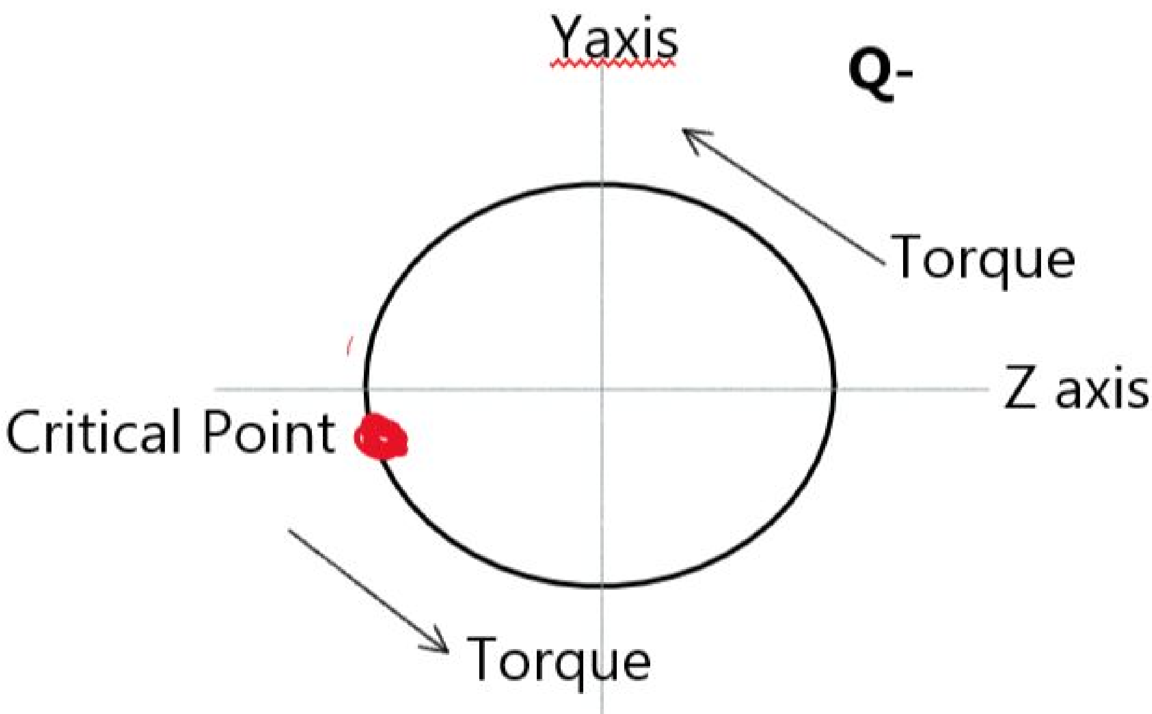
$$xy\ plane \sum Mp = 0 = \mathrm{RAry} * (\mathrm{L} - \mathrm{a}) - \mathrm{RAly} * \mathrm{a}$$

$$xz\ plane \sum Mp = 0 = \mathrm{RArz} * (\mathrm{L} - \mathrm{a}) - \mathrm{RAlz} * \mathrm{a} - \mathrm{Fpulley} * (\mathrm{a} + \mathrm{c})$$

Peak Shear. Moment, and Torque:
Peak Torque = Ti = 88.67 Nm = TA
Peak Moment = PulleyForce * (a +c) = 119.42 Nm = MA
Peak Shear = Largest Force on the Shaft = PulleyForce = 3.040 kN

c) Critical Cross Section and Most Critical Point

The critical cross section is located at the peak bending moment, at section Q- (shown in the diagrams on the first page of this appendix. The most critical point is where the stresses are highest. Q is the location of peak moment. Q- still has a torque acting, which makes it more critical than Q+. The critical point is labled below



d) Stresses and Dynamic von Mises Stresses

Allowable Stress Factors
k0 = 1
Kf = 0.286 from ground surface finish
Ks = 0.790
Kr = 0.82 for 99% reliability
Kt = 1 for no temperature factor
Km = 1 for no other miscellaneous factors

Material and Shaft Properties:
Chose 12L14 Carbon Steel to use for the shaft, it is cheaply available on McMaster
Sut = 540 MPa (material ultimate tensile strength)

Sprimee=0.5*Sut = 270 MPa (.5 for bending predominantly)
SeA=ko*kf*ksA*kr*kt*km*Sprimee = 50.04 MPa
Area of shaftA = pi * dA^2 * .25 = 0.0012 meters squared

Stresses:

sig_A = 32*MA/(pi* (dA^3)) = 21.482 MPa ; Also the amplitude stress for computing von Mises calculation

sig_m = 0; Axial stress is zero due to lack of axial forces (Spur gears were chosen because they are cheaper/ easier to manufacture). Also midrange stress for computing von Mises calculations

t_mA = 16*TA/(pi * (dA^3)) *.5 = 3.988 MPa; Midrange Torsional Shear for computing von Mises calculations

t_aA = (4 * VA) /(3 * areaA) = 3.500 MPa; Amplitude Torsional Shear for computing von Mises calculations

sig_vm_midrangeA = sqrt (3*(t_mA)^2) = 6.907 MPa; Midrange von Mises

sig_vm_amplitudeA = sqrt ((Kf*sig_A)^2 + 3*(Kfs*t_aA)^2) = 44.642 MPa; Amplitude von Mises

e) Factor of Safety from Goodman Line

Safety Factor of Shaft A:

SFA = (sig_vm_amplitudeA/SeA + sig_vm_midrangeA/Sut) ^-1 = 1.105

2. Analysis of Gear 1 of Shaft A

a) Gear Dimensions, Forces, Speed, Expected Life

Gear Parameters and Dimensions:

Gear teeth sizes were chosen after using a matlab code to evaluate possible values of gear teeth and optimizing the numbers of teeth to improve the safety factor on each shaft

N1 = 21 teeth in gear 1

phi = 14.5 degrees, the pressure angle of the gears (value commonly used by gears available on McMaster Carr)

m = 4 mm , the module of gears 1 and 2 (common value which passed factor of safety checks for the shaft and gear teeth)

d1 = 84 mm, the diameter of gear 1 (d = m*N1)

bw = 8*m = 32mm, the face width of gear 1, 8 times the module (the minimal value was chosen for a compact and shorter shaft which is stronger and more material efficient)

Gear Forces on Shaft A:

Ft1 = 2 * T1 / d1 = -2.111 kN Force on the gear in the Z axis

Fr1 = Ft1 * tan(phi) = -0.546 kN Force on the gear in the Y axis

Gear Speed:

nA = 2800 rpm Speed of shaft A is the given input speed

Expected Life:

Expecting the system to operate for 5 years, 5 days a week, 52weeks a year, and 8 hours a day results in a lifetime of 624,000 minutes. With the given input speed, the number of load cycles can be computed

NA = 1,747,200,000 cycles

b) Materials, Hardness, Strength Factors, Strength Analysis

Material and Hardness:

Gear 1 is smaller than the gear it mates with, gear 2. Thus it is a pinion, and should be harder than the mating gear as it endures more load cycles. A brinell hardness of 400 was chosen, a Grade 2 steel as mentioned in Chapter 5 of the class text.

poisson = 0.28 ; poisson's ratio for the chosen steel

E = 210*10^9 ; young modulus for the chosen steel [GPa]

Strength Factors:

sb_1 = 0.703*HB13 + 113 = 394.2 MPa ; allowable bending stress before modification

sc_1 = 2.41*HB13 + 237 = 1201 MPa ; allowable contact stress before modification

kr = 1 reliability factor for 99% reliability

kt = 1; %temperature factor, = 1 for T<125 degrees C

ynA = 1.6831 * NA ^-0.0323 = .8464 bending stress cycle for shaft A, lower curve from textbook Figure 5-46 for more conservative estimate

znA = 2.466 * NA ^-0.056 = .7489 ;contact stress cycle for shaft A

A' = 0.007

sig_1bend_all = (sb_1*10^6 * ynA) / (kt * kr) = 333.66 MPa ; allowable bending stress for gear element 1

sig_1contact_all = (sc_1*10^6 * znA * cHG_2) / (kt * kr) = 907.80 MPa ; allowable contact stress for gear element 1

c) Stress Factors and Stresses

Stress Factors:

kb = 1 ; rim thickness factor

ki = 1 ;idler factor, no idler gears
km = 1 ; load distribution factor
ks = 1 ; size factor
ka = 1 ; application factor, equals 1 because the electric motor input is smooth
ke = sqrt (1 / ((1 - poisson^2)/ (E))) = 4.7735 * 10^5 ; elasticity factor

<u>Dynamic Factor:</u>
q = 6; %high end commercial gear
B = ((12 - q) ^ (2/3)) / 4 = 0.8255
A = 50 + 56 * (1-B) = 59.773
v1 = pi*nA*m*N1/60 = 12.315 ; velocity of a tooth on gear 1
kv1 = ((A + sqrt (200*v1))/ A) ^B = 1.647 ; dynamic factor for gear 1
I1 = pi * cos(phi) * sin(phi) / (1 + N1/N2)
Y1 = .24

<u>Stresses:</u>
sig_b1 = wt12 *ka * ks *km *kv1 * ki * kb / (bw * m * Y1) = 113.9 MPa ; tooth bending stress
sig_c1 = sqrt (wt12 *ka * ks *km *kv1 / (bw * m*N1 * I12)) *ke = 743.6 MPa ;tooth contact stress

d) Safety Factors

SF1B = sig_1bend_all/sig_b1 = 2.948; Bending Stress Safety Factor
SF1C = sig_1contact_all/sig_c1 = 1.221; Contact Stress Safety Factor

3. Bearing Selection

a) Reaction forces

<u>Bearing Forces on Shaft A;</u>
Y and Z axis components of the Bearing Forces (radial and thrust respectively). Note the naming convention, RA corresponds to the reactions on shaft A. “l” and “r” correspond to the left and right bearings when looking at the drawings of the shaft assembly. “y” and “z” correspond to the forces along the Y and Z axis.

RAly = (-Fr1 - (1+(a+c)/(L-a))*Fpull*sin(theta)) * ((L-a)/L) = 0.366 kN
RAlz = (-Ft1 - (1+(a+c)/(L-a))*Fpull*cos(theta)) * ((L-a)/L) = -2.601 kN
RAry = -RAly+Fr1+Fpull = 0.180 kN
RArz = -Ft1 * (a/L) = 1.672 kN

<u>Resultants of the Bearing Forces:</u>
RRAl = sqrt(RAly^2 + RAlz^2) = 2.627 kN
RRAr = sqrt(RAry^2 + RArz^2) = 1.682 kN

b) Bearing type

Choosing a single row deep groove ball bearing from SKF 6 series product line. It is ideal for the high speed, and significant radial forces which the bearing must accept. It will also operate with low power loss and simple maintenance.

c) Equivalent loads, lifetime expected

With the expected life of the shaft in minutes (from 2-a) and the shaft speed, the L10 life for the bearings on shaft A can be calculated
L_{10} = 624,000 * nA / 10^6 = 1,747.2

The bearing encounters pure radial loads, which are equivalent to the largest resultant bearing force, the one on the left side in this case, RRAl = 2.627 kN = P

d) Calculated C

C = PL_{10}^⅓ = 23.836 kN

e) Bearing selected, ID, OD, width

<u>Bearing Selected:</u>
Using table 6-3 from the class textbook, **SKF 6208** meets the load requirement (can handle up to 30.7 kN). This bearing will not need to be changed during the expected lifetime of the gearbox.

<u>ID:</u> 40 mm (compatible with shaft's minimum diameter)

<u>OD:</u> 80 mm

<u>Width:</u> 18 mm

4. Pulley Consideration

There were three main factors to consider for the pulley application element; the diameter of the pulley element, the ratio of the belt tensions, and the orientation of the belts in the YZ plane.

a) Pulley Diameter

dP = 350 mm,
A large diameter is desired to lessen the stresses on the shaft. It also reduces the magnitude of the bearing reaction forces. A larger pulley is likely to be much cheaper than a larger bearing and larger diameter shaft. This diameter was chosen to be around the same size as the largest gear in the gearbox assembly.

b) Belt tensions

Fpull1/Fpull2 = 1.4 ; the ratio of tension from tight over slack sides of the pulley belt

A higher ratio increases the safety factor of the shaft. Too high of a ratio is unrealistic for the friction applied to the pulley

Fpull2 = 2*Ti/(dP * (Fpull1/Fpull2 - 1)) = 1.266 kN ; tension in the slack side

Fpull1 = 1.5 *Fpull2 = 1.773 kN ; tension in the driving side

Fpull = Fpull1+Fpull2 = 3.040 kN ; total force applied on the shaft from the pulley

c) Belt Orientation

The pulley was oriented so the sum of the belt tension forces acted along the positive Z axis. This orientation was chosen after running through possible orientations in the YZ plane in MATLAB. It was found that applying the force along the Z axis is most effective. The angle varied from 4-10 degrees depending on the size of the pulley, but in the interest of easy connection to the motor, an angle of 0 along the Z axis, was chosen.